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Energy Balance In The WTC Collapse

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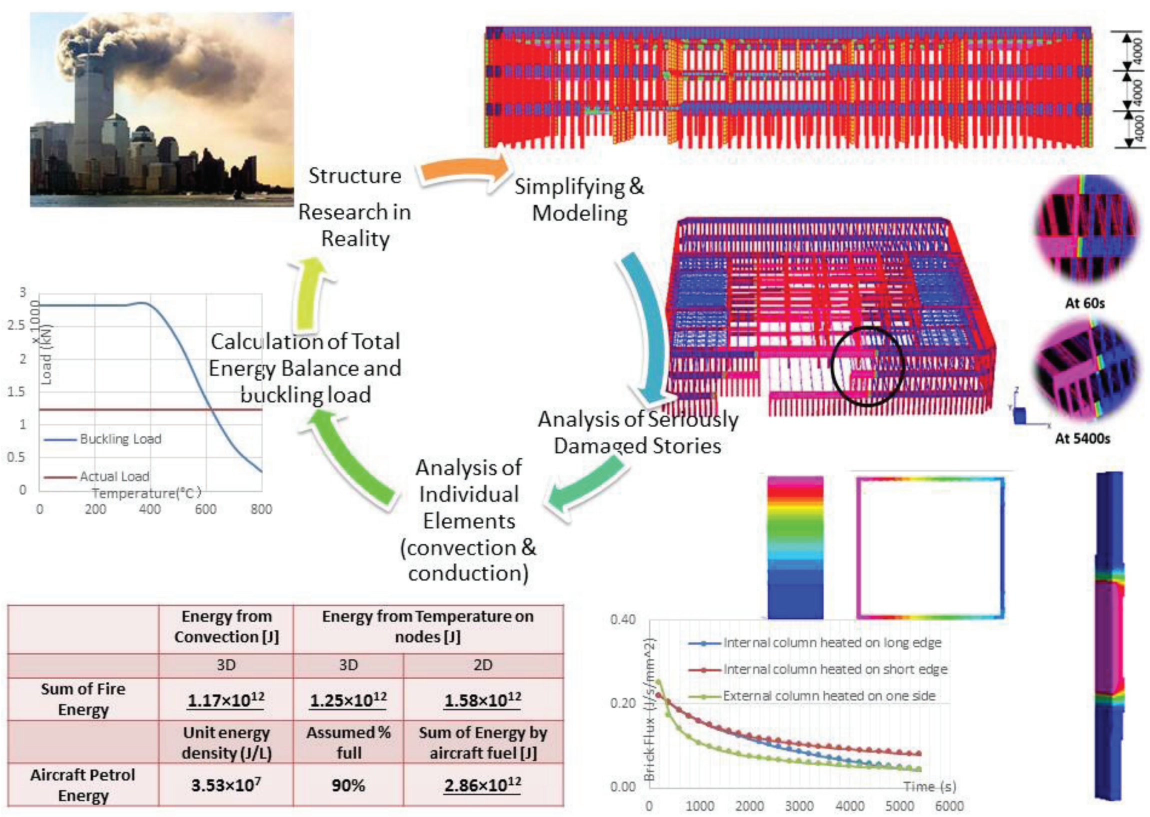
Abstract. The main aim of this report is to provide an analysis of Twin Towers of the New York City's World Trade Centre collapsed after attacked by two jet aircrafts. The approach mainly focused on the effect of temperature on mechanical properties of the building, by modelling heat energy in the south tower. Energy balance during the collapse between the energy inputs by aircraft petrol and the transient heat to the towers was conducted. Both the overall structure between 80 to 83 stories and individual elements was modelled. The main elements contributed to the heat transition includes external and internal columns. Heat applied in 2D and 3D models for single elements was through convection and conduction. Analysis of transient heat was done using Strand7.



	Energy from Convection [J]	Energy from Temperature on nodes [J]	
	3D	3D	2D
Sum of Fire Energy	1.17×10^{12}	1.25×10^{12}	1.58×10^{12}
	Unit energy density (J/L)	Assumed % full	Sum of Energy by aircraft fuel [J]
Aircraft Petrol Energy	3.53×10^{7}	90%	2.86×10^{12}

Graphical abstract. Heat flow within the three-level model at 60s and 5400s and elements including internal and external columns and truss are attached.

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INTRODUCTION

Framed Tube System Tower with multistories

Twin Towers were components of the World Trade Centre located in Lower Manhattan, New York City. They consisted of two 110-story with 6-level basement commercial office buildings: the 417 m North Tower, and 415 m South Tower. At 9:03 a.m. on September 11 in 2011, the Twin Tower collapsed after attacked by two jet aircrafts. This project was aimed to give a transient thermal analysis of the tower element in this process and a calculation of the entire energy flow in the building. We focused on the South tower, it collapsed within 1.5 hour after the attack. As a framed tube structure, it occupied approximately 63m x 63m with core of roughly 27m x 41 m (detailed dimensions shown below). There were 59 external columns on each side of the structure and 4 columns on the four corners, thus there was a total number of 240 external columns. The structure core consisted of 47 steel columns running from the bedrock to the top of the tower. The large, column-free space between the perimeter and core was bridged by prefabricated floor trusses. Trusses in between connected the core to perimeter wall with a spacing of 2.03 m centre to centre.

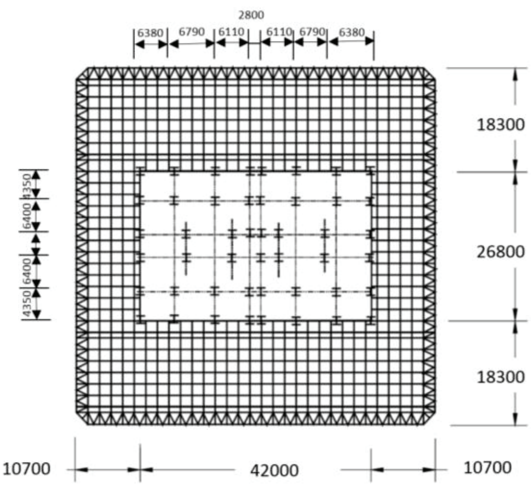


FIGURE 1. Structure Introduction: plan view of WTC (unit mm)
[Federal Insurance and Mitigation Administration, 2002.]

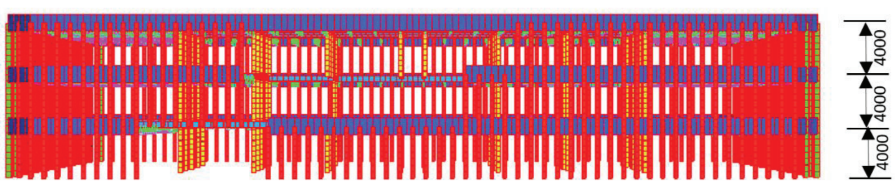


FIGURE 2. Structure Introduction: Elevation view of WTC from level 80 to 82(dimension only refers to the strand7 model, for exact level heights for entire building please refer to the appendix)

This project aimed to investigate the temperature transfer and energy flow within the building after the attack till collapsing.

TABLE 1. Building Details

Location: New York City	**The Year of Built:** 1968
Architects: Yamasaki & Associates	**Approximate Cost:** $450 million
Structural Engineers: Lesile E. Robertson Associates	**Overall Height:** 415m
Function: Commercial office	**Floor Area:** approximately 3,700 m^2
The Structure of the Plan: A square shaped floor space around an square core	
Number of Floors: 110 floors above ground, and 6 levels of basement	

STRUCTURAL MEMBERS

Below is the description of the members of the structure:
Perimeter walls: The perimeter wall was consisted of eexternal Vierendeel trusses columns. The size of a single square- hollow-section external column was 365mm×365mm, with thickness varies from 6.35mm to 63.5mm. Three columns spaced in 1016mm are connected together by a 1320mm wide spandrel, and forms a piece of the perimeter wall (about 10m in length), as shown below. To ease the design, all of them was modelled in same size of 365mm×365mm, with thickness of 9.5mm.

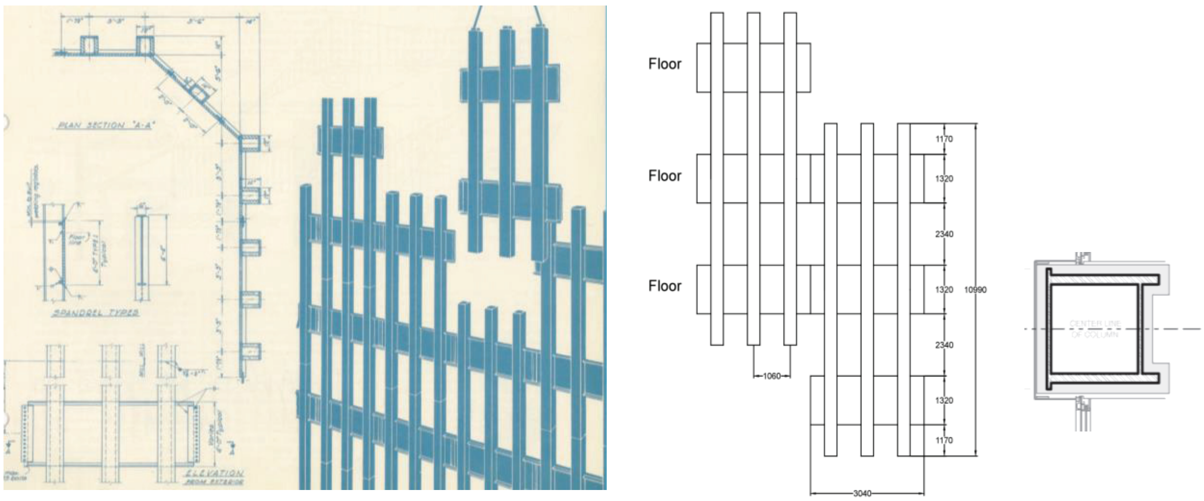


FIGURE 3. Schematic of contemporary steel design – perimeter columns with corss section (unit: mm) [American Iron and Steel institute, 1964]

Core columns: Core of the framed tube was mainly supported by core columns, varied in both dimensions and shape. For lower floors of the structure, core columns were exclusively large box columns of roughly 300mmx1320mm, as the original design of 178mm thickness was not accommodated. From ground to 66 floors, there were 47 steel columns, among which 12 columns were 1400mmx560mm and 35 columns were 600mm x 460mm in size. For upper stories, some columns were replaced by an I-steel and for stories above 84 floors, all of them were I-shaped columns. Length of columns depended on the storey height varied from 3050 to 6710 mm (assumed a uniform length of 4000 mm when modelling).

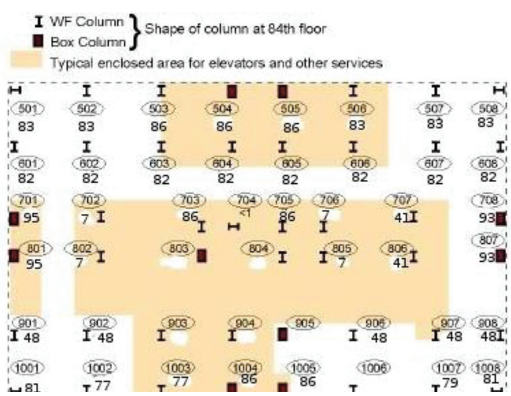


FIGURE 4. Structrual plan view of core column in WTC towers on a typical level (Floor 84) [Federal Emergency Management Agency, 2002]

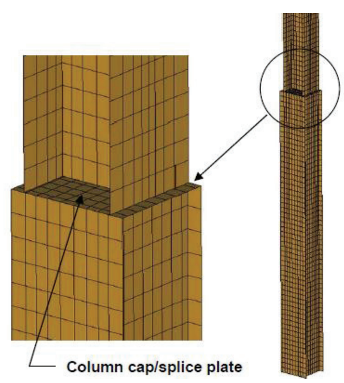


FIGURE 5. Detains of box column-to-wide flange core column splice [Federal Emergency Management Agency, 2002]

Trusses: Trusses welded to the core and connected by 25mm diameter bolts to the external columns were spaced 2.03m both in transverse and longitudinal directions. They spanned 10700mm or 18300mm depending on the distance from the core to the edge of the tower and the main trusses were always in the longer span. It consisted of two layers of steel plates of roughly 100mm thick, beneath a fireproofed concrete deck. The top and bottom steel plates were 900mm apart and connected by a 28mm diameter cable.

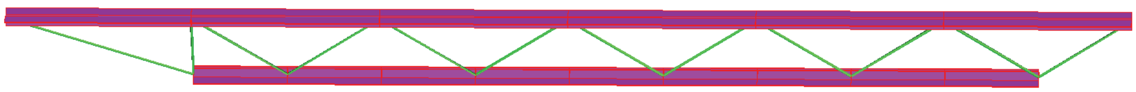


FIGURE 6. Trusses element in Strand 7 structural model

TABLE 2. Structural Element for the Level 1 (Building – Projects)

Details of the Structural Elements	**Structural Element Sizes**
Columns - Structural Systems	External:365*365*9.5
	Internal:300*900
Trusses - Structural Systems	1800*740

LOAD

Load applied to each column was calculated from data in reference 5. Assuming the core supported 53% of the buildings weight and the perimeter columns supported 47% referring to the NIST (National Institute of Standards and Technology) NCSTAR1 report. The load applied on each column was calculated by adding all the dead loads and live loads above level 83, dividing by the number of columns. Dead load was the sum of the weights of construction materials and the permanent non-varying loads from non-structural components such as wiring, plumbing, heating and cooling aggregates and elevators. Live load was assumed to be a quarter of maximum design loads. For more detailed calculation please refer to the appendix.

TABLE 3. Load Capacity and Applied Load on Columns

	Load Capacity	**Applied Load**			
	Buckling Load Capacity (kN)	**Dead Load (kN)**	**Live Load (kN)**	**G + Q (kN)**	**1.2G + 1.5Q (kN)**
External column	2817	912	327	1239	1585
Internal column	57079	3059	645	3705	4639

TEMPERATURE

The maximum flame temperature of hydrocarbons (jet fuel) in air is 1000^{0}C. Consider to the diffuse flame and other energy loss, the fire temperature was set at 800^{0}C. Though steel would melt at 1500^{0}C, 800^{0}C was sufficient enough for steel to lose more than 90% of its bearing capacity and cause the failure of a structure. Hence, it is realistic to to assume the fire temperature as 800^{0}C, and the initial temperature of the whole structure as 20^{0}C.

The iso-fire diagram on buildings was considered in the first place. However, in iso-fire diagram, it was assumed that the temperature was increased from 0^{0}C and hence not suitable to the project with actual initial temperature f around 20^{0}C. Additionally, due to the uncertainty in thermal property of building materials and the limitation of fire temperature, it was not applicable to apply the diagram to this project in a suitable way. Thus the properties in iso-fire diagram was not adopted.

NUMERICAL ANALYSIS

Strand7 was used for the finite element numerical analysis. Two types of model were created to stimulate the heat flow. Structural model was utilized to analyses how heat propagated alone the entire steel structure, meanwhile, the detailed element models would illustrate the temperature change as well as the energy flow in the affected elements. In this project, only Transient Heat solver was used for all models, and the period was chosen as 1.5 hours (5400s) ("Collapse Of The World Trade Center") which was the average time between collision and collapse of two buildings. Thermal material properties were assumed temperature independent and the problem only involved conduction and convection, consequently, only liner heat solutions were required.

For the structural model (shown in Fig. 7), considering the limitation in computational time, only four floors (80th to 83rd floors) struck by airplane were modelled. The beam elements were used in this structural model, the cross-sectional dimensions of each element were set in element property. As it was a steel structural building, Structural Steelwork (AS 4100-1998) was chosen as the material for all, concrete cover was neglected. Damaged parts from collision to the building were simplified as a hole, within the region, all the elements were removed. A fixed 800^0C (Eagar and Musso 8-11) was applied to every element in contact with the surface of the plane-shaped hole left by the aircraft impact, in order to simulate the ignition.

While modelling the detailed elements, as the floors and walls were either protected by concrete cover or fireproof layers, considering the complexity and uncertainty of the material thermal properties, only steel columns were analysed. The energy flow in air was also neglected. Both two dimensional (2D) and three dimensional (3D) models were utilized, and the material property for all models were Structural Steelwork (AS 4100-1998) provided in Strand7.

In 2D models, plate element was used to create the cross section of both internal and external columns, the thickness of which was set to 4m in geometry to simulate the column height. Since the cross section of internal columns was rectangular, to be more precise, a fixed 800^0C was applied to nodes at either long edge or short edge to do the analysis separately, with initial nodal temperature of 20^0C at the other nodes.

Three connected columns were created with brick elements in 3D models with the middle one exposed to fire only, in order to obtain both heat flux and energy propagation along the columns. In 3D models, not only nodal temperature, but also air convection was considered. Similarly, the analysis was done by applying fixed 800^0C to the surface nodes at either long edge side or short edge side along the column length. The rest of the nodes were set to initial 20^0C. Under convection condition, instead of applying fixed nodal temperature, the ambient temperature at the surface of the brick element in contact with fire was defined as 800^0C. Because the free convection coefficient for air, gases and dry vapor varies from 0.5 to 1000 (W/(m^2C) ("Convective Heat Transfer")), to be more conservative, the hc=1000(W/(m^2C)) was applied to the same brick surface together with the ambient temperature. The initial temperature of 20 ^{0}C was added in Load Case 1 and involved in Transient Heat solver.
In transient heat analysis, the governing equation is (based on Fourier law):

$$\nabla^T \mathrm{q} + \rho \mathrm{C} \frac{\partial T}{\partial t} = Q \quad (1)$$

Where q is the heat flow, ρ is the material density, C is the specific heat, T is temperature and Q represents the heat energy generated per unit of volume. In conduction analysis for both 2D and 3D models, the boundary condition is,

$$\mathrm{T} = T_{ref} \quad (2)$$

When it comes to convection, the boundary condition is,

$$q^T n = h_c (T - T_{ref}) \quad (3)$$

Where T_{ref} is the ambient temperature.

RESULTS AND DISCUSSIONS

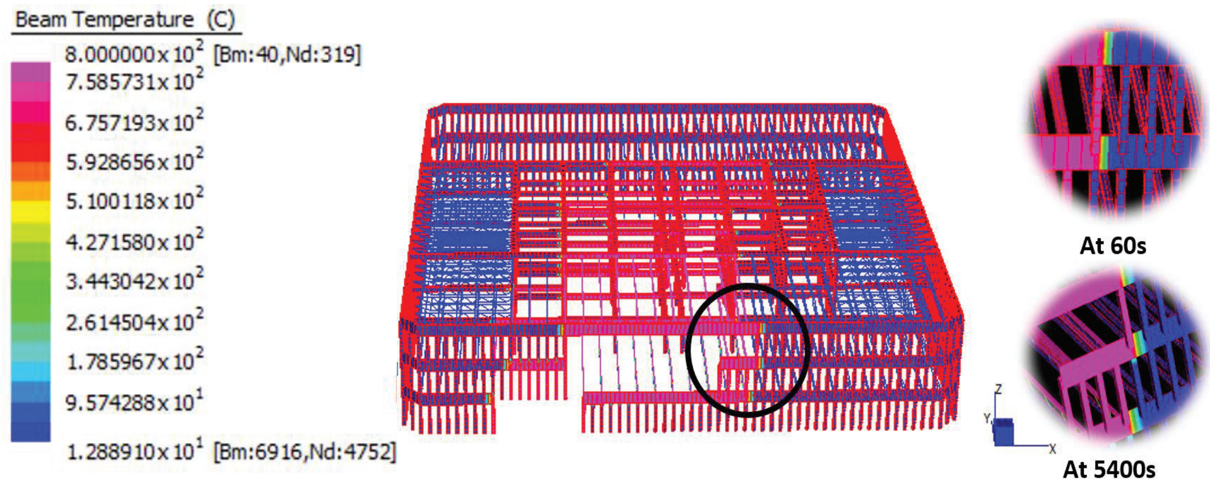
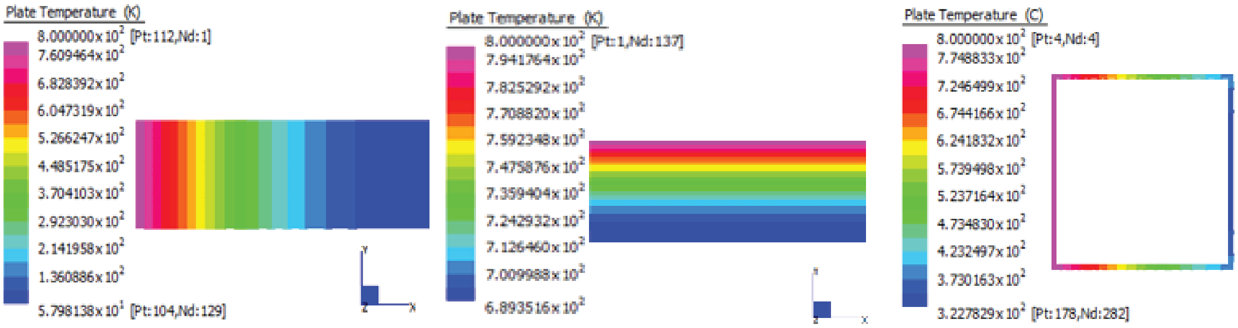


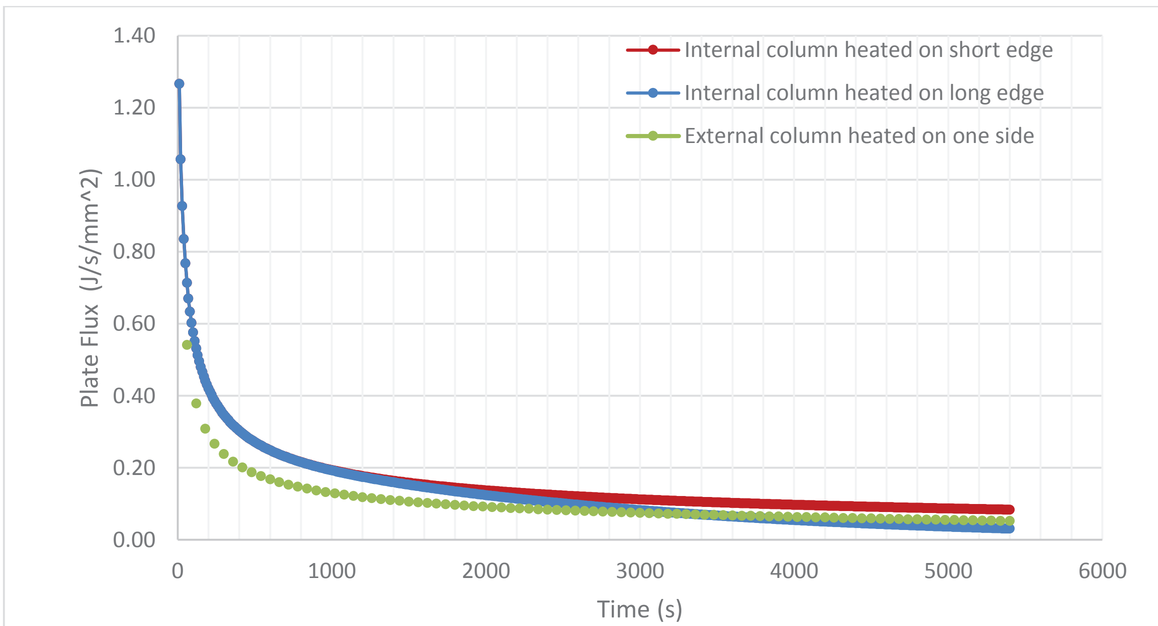
FIGURE 7. Result of temperature transfer after 1.5 hour using 3D model analysis (from level 80 to 82 of WTC)

Figure above shows the result of the structural model after burning for 1.5 hours. Temperature was illustrated by different colours as shown in the chart on the left. The temperature of the entire model was presented by colours on the right, with details of a typical connection (circled part) burning performances at 60s and 5400s. It can be observed that the temperature only changed a little along the element, which shows the heat propagation on steel was very slow. Along the connecting beams between two external columns, one metre away from the heat source column, the temperature was only around 200^{0}C after 1.5 hours. As steel would only change its bearing capacity at 700^{0}C or above, the result of 200^{0}C temperature increase would hardly change steel`s property, which means the adjacent column was barely affected by the heat diffusion through structural elements only. According to this, the capacity reduction mainly happened to elements in contact with the heat source, and the energy was mainly absorbed by those elements as well.

2D Temperature at node model results



(a) (b) (c)



(d)

FIGURE 8. (a) Temperature applied on the long edge of internal column; (b) Temperature applied on the long edge of internal column; (c) External column heated on one edge; (d) Flux in columns

3D Temperature at node model results

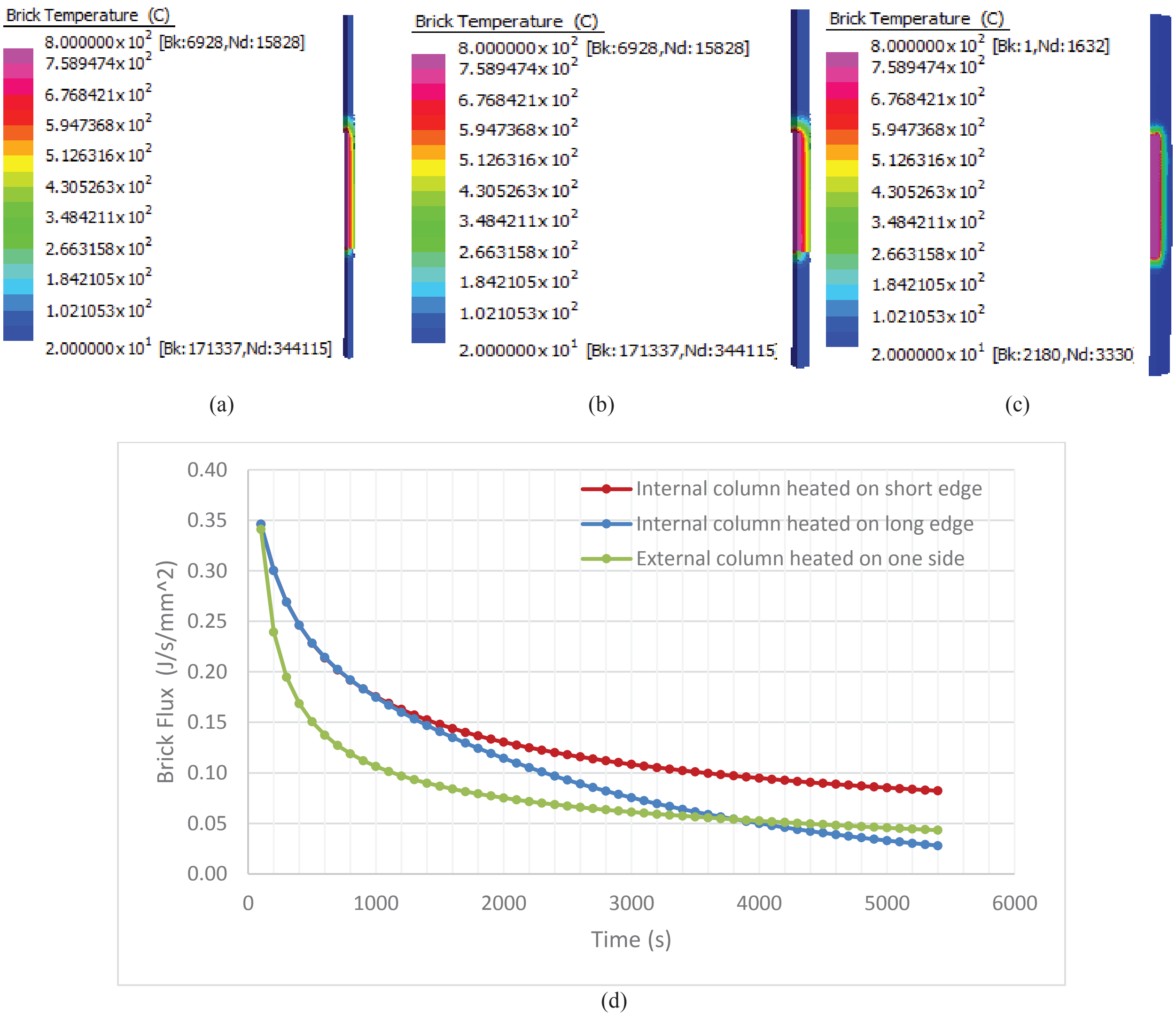


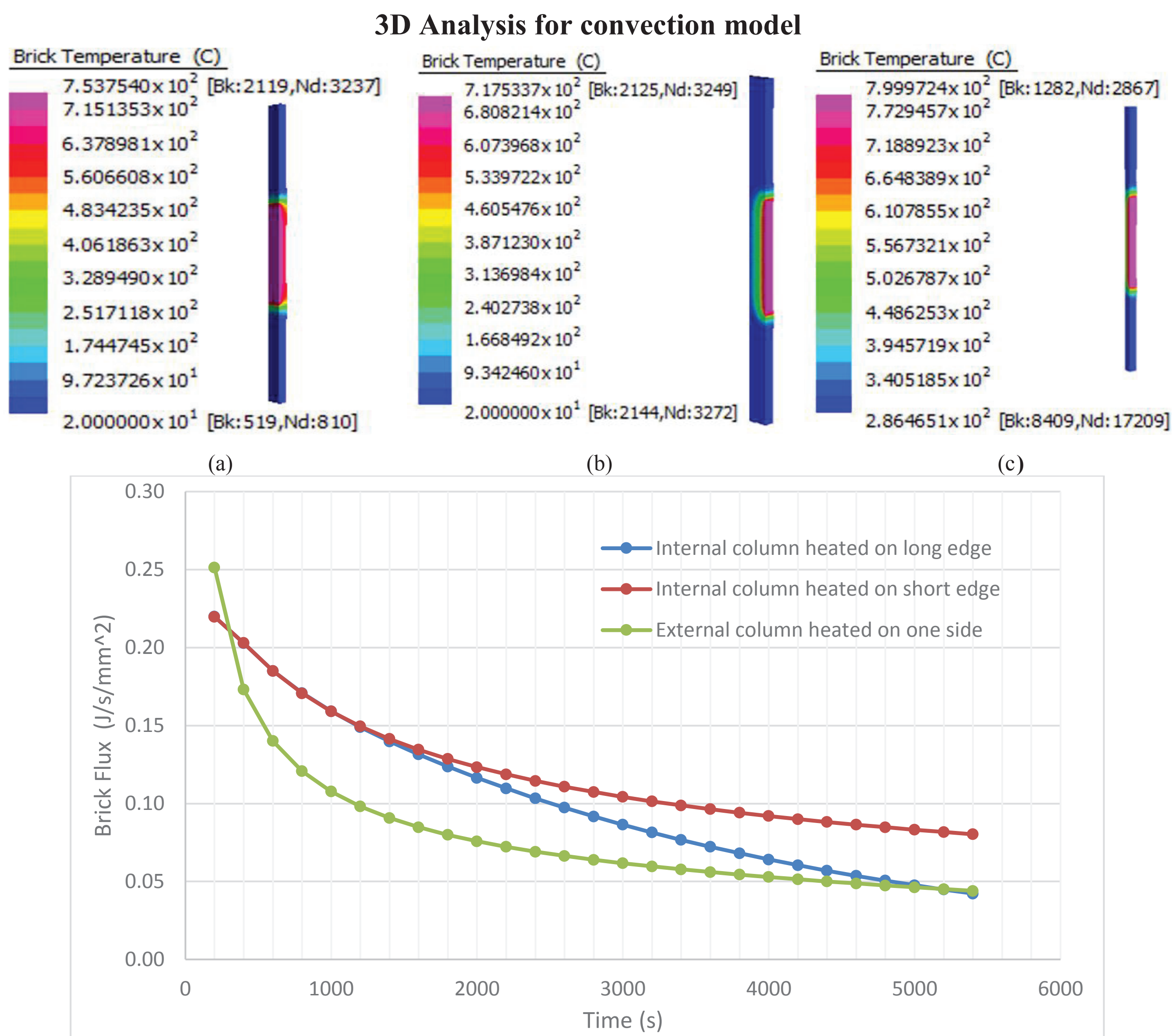
FIGURE 9. (a) Temperature applied on the long edge of internal column; (b) Temperature applied on the long edge of internal column; (c) External column heated on one edge; (d) Flux in columns

Results of heat conduction within column cross sections and between each floor during the fire due to the plane fuel leakage were done in 2D (Quad 4 elements) and 3D (Tetrahedral elements) models as shown above. The heat transits from the heated face to its adjacent sides then across the entire column member in the direction where the heat applies, or spread to its adjacent columns from middle column outwards. For both 2D and 3D models, the temperature of face on fire was assumed to be fixed at 800°C. Thus the highest temperature achieved is about 800°C for each model. For 2D models, the lowest temperature on the cross sections is around 60°C and 300°C, for core columns and external columns respectively. For 3D models, the lowest temperature stays at its initial value of 20°C, on the ends of upper and lower columns.

For flux analysis, generally, energy absorbed by core columns is higher than energy absorbed by external columns. In 3D models a smaller flux results generated than in 2D models. For interior columns, flux absorbed when heat applied on shorter edge of column is smaller than that when heat applied on longer edge of column. This may due to that for limited small area surface, the capacity of it to absorb the heat may be smaller than that for a large area surface. Thus the flux is lower and slower spread across the column surface with shorter edge. Similarly, in external columns, the face area is smaller than the face area of long edge core column surface, thus the heat absorbed by external column face is smaller than the larger core column face. Moreover, as the external columns are hollow sections, the energy it can absorb is less significant compared to the solid internal columns.

In 3D models, the direction of heat transient is in 3 directions, which consumes longer time for heat to transfer compared with 2D models. Furthermore, when Strand7 plot the flux graphs it assumes the flux can only transit in one direction where heat applies and assumes no flux in other two directions. In reality the flux goes everywhere and thus the result from only one direction of flux may have been reduced. For 2D models the flux only goes in one direction and the result is not deducted hence larger. These may explain the reason for that 2D element flux is greater than 3D element flux.

Though there are some discrepancy between them, the results are overall very similar to each other.



(d)

FIGURE 10. (a) Internal column heated on the long edge; (b) Internal column heated on the short edge; (c) External column heated on one edge; (d) Flux in columns

The convection analysis provided a more realistic comparison. In actual situation, fire burnt on jet fuel, the generated heat propagated to the structure via convection. Therefore, there were two stages of energy consumption, energy absorbed by air and transferred from air to structural elements.
From the results, after 1.5 hours, external column had the highest temperature among all which was nearly 800°C. This was due to its hollow cross-sectional geometry. According to the heat capacity of a body with constant volume, for temperature change only,

$$C_v = \frac{\Delta Q}{\Delta T} \tag{4}$$

Where C_v is the heat capacity at constant volume, which is different from the C value in Eq. (1), ΔT is the change in temperature and ΔQ is the amount of heat transferred. Compared with solid column (such as the internal column), the amount of steel volume reacting on convection was less. The difference in the amount of energy provided by air during a same period was neglected compared with the difference in volume. As a result, the increase in temperature was greater than internal columns. For internal columns, the one with long edge exposed to fire ended up with higher temperature. Since the interacting surface was three times bigger, it had more chance to absorb energy from the air.

According to the flux graphs, for all members, the flux decreased with time but the trend was getting smaller. Heat flux in multi-dimensional case is

$$\vec{q} = -k\nabla T \tag{5}$$

Where k is the coefficient of conductivity , T is temperature and ∇ is the gradient operator. The element temperature was increasing by time, therefore, the temperature gradient was reducing caused the decrease in flux. In this project, flux represents the speed of energy assimilation. Reduced flux would slow down the ascending in temperature. That could explain why the temperature changed fast at the beginning but slow in the final stages. Additionally, the highest temperature for each model did not reach 800°C This was also due to the feature of convection. The interaction between steel and air were becoming less active by time and getting more stable in the end. If the time period was long enough, the element temperature would probably become 800°C.

Results Summary

The heat energy could be calculated from the integration of flux and time, times the area of heated faces:

$$\text{Energy} = \text{A}\cdot \int_0^{5400} \emptyset(t)dt \ [\text{J}] \tag{6}$$

Where $\emptyset$ is flux per second per mm^2, A is the area of heated column face in mm^2; the kinematic energy on one face of column starts from 0 to 1.5 hours was calculated (results shown in the following section).

TABLE 4. Energy results per surface of column

		Face area [mm^2]	**Heat Energy**	**Model heated by Convection**	**Model heated by Temperature on nodes**	
				3D	3D	2D
Core column	Heat on short edge	1.20E+06	$\int_0^{5400} \emptyset(t)dt$ [J/mm^2]	6.20E+2	7.02E+2	8.48E+2
(300×600mm^2, solid)			Energy [J] /face A·$\int_0^{5400} \emptyset(t)dt$	7.44E+8	8.42E+8	1.02E+9
	Heat on long edge	3.60E+06	$\int_0^{5400} \emptyset(t)dt$ [J/mm^2]	5.35E+2	5.60E+2	7.20E+2
			Energy [J] /face A·$\int_0^{5400} \emptyset(t)dt$	1.93E+9	2.02E+9	2.59E+9
Perimeter column	Heat on edge (same)	1.46E+06	$\int_0^{5400} \emptyset(t)dt$ [J/mm^2]	4.04E+2	4.26E+2	5.39E+2
(365×365mm^2, ×9.5mm thk)			Energy [J] /face A·$\int_0^{5400} \emptyset(t)dt$	5.90E+8	6.22E+8	7.87E+8

TABLE 5. Total energy based on assumption

NO. of faces on fire per column	Type of column		No. of Columns	Energy from Convection [J]	Energy from Temperature on nodes [J]	
				3D	3D	2D
Four	External		180	4.25E11	4.48E+11	5.67E+11
	Internal		34	1.82E11	1.94E+11	2.45E+11
Three	Internal	2×Short+1×long	43	1.47E11	1.59E+11	1.99E+11
		2×long+2×short	86	3.95E11	4.19E+11	5.33E+11
One	External		44	2.60E10	2.74E+10	3.46E+10
			SUM	**1.17E+12**	**1.25E+12**	**1.58E+12**

The aircraft fuel energy could be calculated by multiplying the unit petrol energy density by assumed fuel volume:

$$\text{Fuel Energy} = U \text{ i } V \text{ [J]} \quad (7)$$

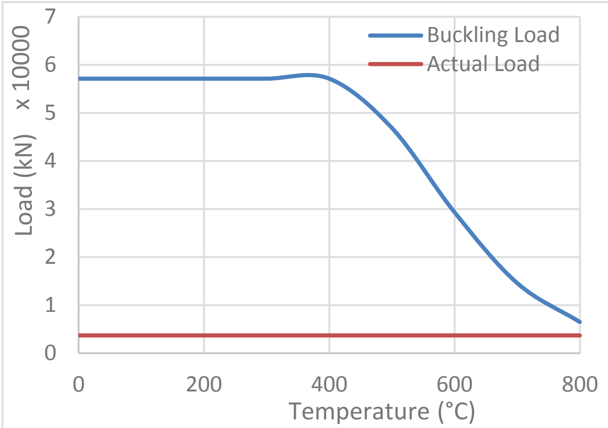
Where U is the unit petrol energy density in J/m3, i is the assumed percentage full of petrol container; V is the total volume of aircraft petrol in m^3 when the container is full (results shown in the following section).

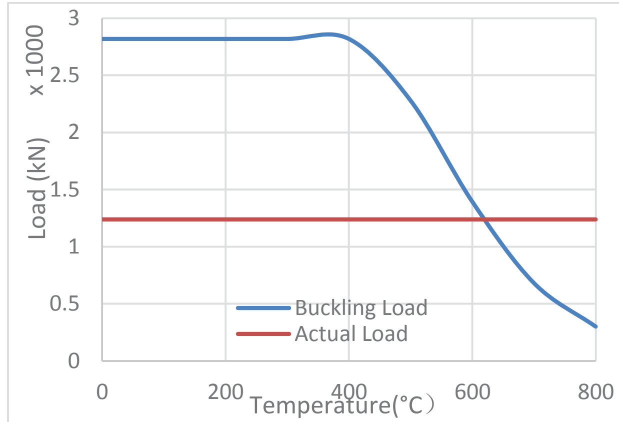
TABLE 6. Energy comparison with aircraft fuel

	Unit energy density (J/m^3)	Volume (m^3)	Assumed % full	Sum of Energy by aircraft fuel [J]
Aircraft petrol	3.53E+10	90	90%	**2.86E+12**

For all elements, the total energy calculated from 3D models under convection was the smallest, while 2D model gave the biggest value. It can be explained by various reasons. In convection process, the column surface was heated up by the air first, which meant the temperature difference between the surface nodes and the other nodes was less than it was in the fixed temperature cases. Referring to the flux equation, it would lead to a reduction in flux, which was also proved by the flux charts above. Consequently, the energy based on the least flux value was the smallest. In 3D models, the heat propagated through x, y, and z directions at the same time. However, only the flux in main direction was captured and analysed. On the other hand, in 2D models, the heat flowed in one direction only, therefore, the flux was fully recorded. So with the same heat source, 3D models should have smaller flux than 2D models.

Compared with the overall energy from the plane fuel (90 m^3 (Eagar and Musso 8-11)), the energy values based on numerical analysis in those three types of models were all a bit smaller than half of the fuel energy. This phenomenon could have been explained in several ways. First of all, the plane flew for a while before it crashed into WCT, therefore, the fuel tank was not full. Secondly, due to the complexity in calculation, heat in the air and surrounding building materials (such as walls and slabs) was ignored. Thirdly, the crashing hole in the building was generated from personal assumptions referring to reality, hence the position and affected area was not that accurate. Additionally, as the fire could travel with air or flue leakage, columns outside of the hole would also be affected, which was not considered in the model. Moreover, the fuel was assumed to be burnt fully, it was hard to achieve in reality due to insufficient oxidizer. Further energy loss in explosion and radiation was excluded as well.
Considering the connection between material property and temperature, the compression capacity of both internal and external columns was calculated based on different temperature conditions. The capacity was compared with the actual loading condition (G+Q) in **FIGURE 11.** (Detailed calculation was in Appendix B.)





(a)Buckling Load in internal column (b) Buckling Load in external column

FIGURE 11. Buckling Load vs Actual Load

It was clear in the chart, the buckling load in internal column was very big due to its solid cross section. With 800°C, when the yielding stress reduced to 10% of it was in room temperature, the compression capacity was close to the actual load but still a little bit greater than the actual load. In reality, the internal column at the collapsing floors were actually with I cross-section instead of solid rectangular cross-section. Hence the bearing capacity should be way less than it was calculated. The internal columns might fail due to the temperature change. However, for external column, the buckling and actual load intersected at around 620°C. As the maximum temperature on external columns was about 800°C after 1.5h, those columns would collapse within this period. As calculated above, the aircraft fuel could provide enough energy to heat the external column up to about 800°C, so it was sufficient to fail the columns. Because the failure of floors caused the collapse of the whole building (Kotsovinos and Usmani 741-765), as the floors were supported by columns only, the heat flow in columns could lead to the collapse of the building.

As the elements` buckling capacity are still larger than the actual loads, the collapse may due to various reasons other than the reduction in element capacity from fire. The tower under attack was experiencing gravity loads, impact load of aircraft head, fuselage cutting force of aircraft wings, wind force and so on. Portion of main structural elements was damaged by the collision that no longer support the upper stories, and the balanced elements` buckling resistance strength was reduced during the fire. Moreover, the fire can burn the office furniture and other flammable materials, and extend to other stories, hence reduce the element strength of the entire building. As a result of all these factors, the levels damaged or on fire cannot withstand the loads, the entire building collapses.

CONCLUSIONS

In this project, the process of WTC Twin Tower collapse was analysed. Influence of heat through structural elements was illustrated through entire structural model and single element models in 2D and 3D. Heat applied on models was tested in two ways through temperature at nodes and convection. In addition, various loads on structure was calculated and compared with the buckling load capacity of elements. The energy mainly transferred into structural elements where exposed to fire as it was found that the propagation of heat through elements was small enough to be neglected. Then, from single element Strand7 results, it was found most of the heat energy from fire was due to the fuel leakage from the airplane. Despite that 3D models give a more precise analysis, the results from 2D model were closer to the fuel energy. This may due to the uncertainty of fire, imperfections in assumption and the limitations of Strand7. The energy from aircraft fuel could cause a significant reduction in column loading capacity therefore led to the buckling of the structure. The whole structure collapsed as a result of both the buckling collapse of columns during the fire and experiencing gravity loads, impact load of aircraft head, fuselage cutting force of aircraft wings, wind force and so on. Consequently, the entire building collapsed.

ACKNOWLEDGMENTS

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APPENDIX A – Actual Floor levels

TABLE 1. Floor height at each level [data from 911 Research Net, access in 2016]

	sub-level no 4-5	**sub-level no 1-3**	**service level 1**	**1-concourse**	**2-6-storage**
Height (m)	3.53	3.05	4.88	6.71	3.53
	7-lower mechanical	**8-upper mechanical**	**9-39**	**40-44 sky lobby**	**45-73**
Height (m)	4.27	3.05	3.66	4.27	3.66
	74-76 upper mechanical	**77 upper esc floor**	**78 sky lobby**	**79-105**	**106 typical**
Height (m)	4.27	3.66	4.27	3.66	4.27
	107 restaurant	**108 lower mechanical**	**109 upper mechanical**	**110 roof to top of roof panels**	
Height (m)	5.33	4.27	3.56	4.67	

APPENDIX B – Load calculations

Buckling load (External column in room temperature):

$A = 13510\ mm^4$, $I_x = 2.85 * 10^8\ mm^4$, $I_y = 2.85 * 10^8\ mm^4$,
E = 200GPa, L = 4m, $f_y = 270MPa$, $k_f = 0.9$

$$r_x = r_y = \sqrt{I/A} = 145mm$$

Nominal section capacity N_s (AS4100 6.2)

$$N_s = k_f * f_y * A = 3283kN$$

Nominal member capacity N_c (AS4100 6.3)

$$\lambda_x = \left(\frac{L_{ex}}{r_x}\right)\sqrt{k_f}\sqrt{\frac{f_y}{250MPa}} = 27 = \lambda_y$$

$$\alpha_b = 0$$

$$\lambda = \lambda_n + \alpha_a\alpha_b = 27$$

$$\eta = 0.0032(\lambda - 13.5) = 0.045$$

$$\xi = \frac{\left(\frac{\lambda}{90}\right)^2 + 1 + \eta}{2\left(\frac{\lambda}{90}\right)^2} = 6.23$$

$$\alpha_c = \xi\left[1 - \sqrt{\left(1 - \left(\frac{90}{\lambda\xi}\right)^2\right)}\right] = 0.953$$

$$N_c = \alpha_c N_s = 3130kN$$

Assume $\varphi = 0.9$,

$$\therefore \varphi N_c = 0.9 * 3130 = 2817kN$$

TABLE 2. Capacity reduction for external columns [Irfanoglu and Hoffmann, pp 62-67]

T	E(Mpa)	fy(Mpa)	φNc(kN)
0	200000	270	2817
100	198000	270	2817
200	180000	270	2817
300	160000	270	2817
400	140000	270	2817
500	120000	216	2277
600	60000	130	1393
700	26000	62	681
800	20000	27	300

Calculations for the internal columns followed the same sequence, and were done by excel. The results were listed in table below.

TABLE 3. Table of results

λ_y	48	λ_x	16	α_b	0	η	0.112	ξ	2.46	α_c	0.87
$N_s(kN)$		72900		$N_c(kN)$		63421		$\varphi N_c(kN)$		57079	

TABLE 4. Capacity reduction for internal columns [Irfanoglu and Hoffmann, pp 62-67]

T(°C)	E(Mpa)	fy(Mpa)	φNc(kN)
0	200000	270	57079
100	198000	270	57079
200	196000	270	57079
300	190000	270	57079
400	190000	270	57079
500	168000	230	46847
600	152000	197	29329
700	128000	97	14606
800	100000	49	6524

Design load per area

TABLE 5. Summary of dead load [Gregory H. Urich, 2016]

	Foundation	**Structural steel /floor:**	**Concrete**	**Superimposed**
DL	4,330 tons	91.6 tons (top)	Above grade (Floor 1-110): 467 tons outside, 242 tons inside core	145 tons above grade
		1,464 tons (bottom)	Below grade (Floor B1 – B6): 1315 tons	7.92 tons below grade
Max. DL pressure	**28.7 kN/m^2**			

TABLE 6. Summary of live load [Gregory H. Urich, 2016]

LL = larger of	Most predominate	¼ the design load
	244 kg/m^2	56,177 tons in sum
Max. LL pressure	4.35 kN/m^2	
Maximum design load	1.2G + 1.5Q = 1.2 x 29.0 + 1.5 x 4.35 = 41.33 kN/m^2	

Design load applied on each column:

TABLE 7. Calculation spreadsheet of design load applied on each column

DL						
Steel	7797	short tons	69365.568	kN		
		Numbers	Loads			
	Ext` col	240	135.8409	kN		
	Core col	47	782.20747	kN		
Concrete	outsiide light concrete					
		Area	28,225	sq ft		
		Thk	4	inch		
		Density	150	lb/ft3		
	thus	Mass	1411250	lb	6277.551	kN
	hence	Ext` col	706.22451	kN	x27 floors	
	insiide normal concrete					
		Area	11,745	sq ft		
		Thk	5	inch		
		Density	150	lb/ft3		
	thus	Mass	734062.5	lb	3265.272	kN
	hence	Int` col	1875.7945	kN	x27 floors	
Superimposed DL	total	4000	short tons	35585.77	kN	
	outside	240	No.	69.68881	kN	
	inside	47	No.	401.2864	kN	
Sum of DL	outside	912	kN			
	inside	3059	kN			
Sum of LL	outside	8830	short tons	327	kN	
	inside	3409	short tons	645	kN	
1G+1Q	outside	**1239**	kN			
	inside	**3705**	kN			